Journal of Computational Science 3 (2012) 460–462

Contents lists available at SciVerse ScienceDirect

Journal of Computational Science

journal homepage: www.elsevier.com/locate/jocs

Modification of algebraic multigrid for effective GPGPU-based solution of nonstationary hydrodynamics problems

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ARTICLE INFO

Article history:
Received 5 April 2012
Received in revised form 23 August 2012
Accepted 27 August 2012
Available online 29 August 2012

Keywords:
GPGPU
Algebraic multigrid
Nonstationary partial differential equations

ABSTRACT

We present modification of algebraic multigrid algorithm for effective GPGPU-based solution of nonstationary hydrodynamics problems. The modification is easy to implement and allows us to reduce number of times when the multigrid setup is performed, thus saving up to 50% of computation time with respect to unmodified algorithm.

1. Introduction

Algebraic multigrid (AMG) [1] is one the most effective methods for solution of large sparse unstructured systems of equations, arising, for example, from discretizations of elliptic differential equations. AMG applies ideas of geometric multigrid (smoothing and correction on coarse grid) to solution of certain classes of algebraic systems of equations. The main advantage of AMG (besides robustness and efficiency) is its ability to solve elliptic partial differential equations discretized on unstructured grids [2]. AMG can be used as a black-box solver for various computational problems, since it does not require any information about underlying geometry. This fact makes GPGPU-based implementation of AMG extremely attractive [3,4].

The algorithm of AMG has two major stages: setup phase and solution phase. The setup phase in classic formulation of AMG is very hard to parallelize because of its intrinsic serial nature. To the contrary, the solution phase allows straightforward parallelization. When GPGPU technique is applied to the solution phase, tenfold acceleration rate is easily achieved [5–7]. However, according to Amdahl's law, we cannot exceed acceleration rate of $3\times$ for solution of single system of equations, since about 30% of computational work belongs to the setup phase. We show in this work that the restriction may be loosened for solution of nonstationary problems with constant or slowly changing coefficients. The improved algorithm was successfully applied to the solution of several hydromechanics problems. In particular, we managed to achieve substantial acceleration rate for several oil reservoir simulation problems.

2. Backgrounds of algebraic multigrid

Multigrid methods have optimal efficiency when solving elliptic partial differential equations [1] and are based on a hierarchy of grids and transition operators. The main idea of multigrid is acceleration of simple iterative method convergence through correction of the coarse grid solution.

Geometric multigrid is based on a predefined grid hierarchy. When solving discretized equation $Au = f$, the grid Ω_l, operator A_l, prolongation operator P_l: $u_{l+1} \to u_l$, and restriction operator R_l: $u_l \to u_{l+1}$ are given on each level l of hierarchy. On every level of the hierarchy single iteration of the multigrid algorithm (V-cycle) consists of the following steps:

1. Current approximation is smoothed with several relaxation steps.
2. Residual $r_l = f_l - A_l\, u_l$ is computed and restricted to the coarser level, where it becomes a right-hand side: $f_{l+1} = R_l r_l$.
3. If level $l+1$ is the coarsest level in the hierarchy, equation $A_{l+1} u_{l+1} = f_{l+1}$ is solved directly; otherwise one V-cycle is recursively applied on this level with zero initial approximation $u_{l+1} = 0$.
4. Approximation on level l is corrected: $u_l \to u_l + P_l\, u_{l+1}$.

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http://dx.doi.org/10.1016/j.jocs.2012.08.008